Nonlinear FDI based on state derivatives, as provided by inertial measurement units

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retour sur innovation

Outline

Nonlinear Fault Detection and Isolation - related work

Fault Detection and Isolation of actuator faults
for Nonlinear control-affine systems

Differential-geometric approach (De Persis & Isidori)

Transform coordinates to design nonlinear residual filters sensitive to faults and decoupled from disturbances.

Differential-algebraic approach (Diop, Bokor, Shumsky...)

Transform the system into a set of differential polynomials, functions of inputs, outputs and their successive derivatives. Use elimination theory to extract fault information.

Inversion-based FDI (Edelmayer, Szigeti...)

Compute the left-inverse to obtain a new dynamical model, outputs = faults, inputs = original inputs, outputs and their successive derivatives.

Objectives

Known drawbacks of those nonlinear methods

- Design of coordinate transforms, tuning of inner parameters
- Successive time derivatives of noisy and disturbed measurements
- Integration of dynamical filters

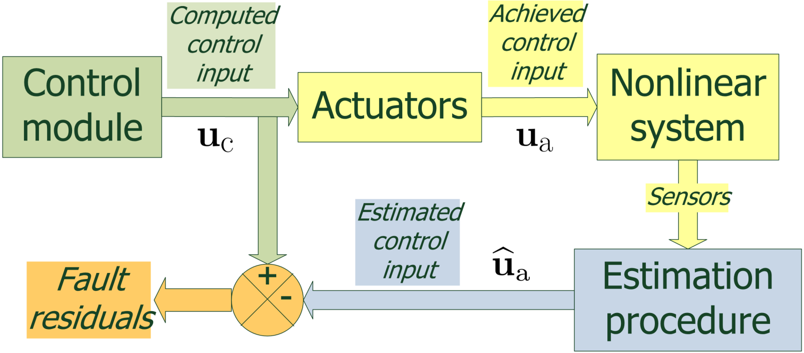
Objectives of the present work

- Avoid numerical differentiation of measured variables
- Avoid dynamical integration, to reduce computational cost
- Assess robustness to model & measurement uncertainty

Means of synthesis

- Take advantage of systems involving *measured* state derivatives (e.g., autonomous vehicles equipped with IMUs)
- Design a completely nonlinear actuator fault diagnosis method

Principles of the approach



Residuals : discrepancy between computed and reconstructed *inputs*

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Planar aeronautical model (longitudinal)

- State vector : $\mathbf{x} = [x, z, v_{\mathrm{bx}}, v_{\mathrm{bz}}, q, \theta]^{\mathrm{T}}$, position, speed, orientation,
- Input vector : $\mathbf{u} = [\delta_m, \eta]^{\mathrm{T}}$, rudder and propulsion
- Measurements : $\mathbf{y} = [a_{\mathrm{bx}}, a_{\mathrm{bz}}, x, z, v_{\mathrm{bx}}, v_{\mathrm{bz}}, q, \theta]^{\mathrm{T}}$, *acceleration*

Nonlinear model

$$\begin{cases} \dot{x} = \cos(\theta) v_{\mathrm{bx}} + \sin(\theta) v_{\mathrm{bz}} \\ \dot{z} = \cos(\theta) v_{\mathrm{bz}} - \sin(\theta) v_{\mathrm{bx}} \\ a_{\mathrm{bx}} = -\frac{Q s_{\mathrm{ref}}}{M} \left[c_{\mathrm{x0}} + c_{\mathrm{xa}} \alpha + c_{\mathrm{x}\delta_{\mathrm{m}}} \delta_{\mathrm{m}} \right] + \frac{1}{M} \left[f_{\mathrm{min}} + (f_{\mathrm{max}} - f_{\mathrm{min}}) \eta \right] \\ a_{\mathrm{bz}} = -\frac{Q s_{\mathrm{ref}}}{M} \left[c_{\mathrm{z0}} + c_{\mathrm{za}} \alpha + c_{\mathrm{z}\delta_{\mathrm{m}}} \delta_{\mathrm{m}} \right] \\ \dot{q} = \frac{Q s_{\mathrm{ref}}}{b} \left[c_{\mathrm{m0}} + c_{\mathrm{ma}} \alpha + c_{\mathrm{m}\delta_{\mathrm{m}}} \delta_{\mathrm{m}} + \frac{l_{ref}}{\sqrt{v_{\mathrm{bx}}^2 + v_{\mathrm{bz}}^2}} c_{\mathrm{mq}} q \right] \\ \dot{\theta} = q \end{cases}$$

$f_{\mathrm{min}}, f_{\mathrm{max}}, M, s_{\mathrm{ref}}, l_{\mathrm{ref}}$ constant parameters. $Q, \alpha, c_{(.)}$ nonlinear functions of $\mathbf{x}$

Preliminary step

- Extract state equations containing control inputs, and involving only *measured* (or *estimated*) state variables and their measured derivatives

$$\begin{cases} a_{\mathrm{bx}} = -\frac{Qs_{\mathrm{ref}}}{M}\left[c_{\mathrm{x0}} + c_{\mathrm{xa}}\alpha + c_{\mathrm{x}\delta_{\mathrm{m}}}\delta_{\mathrm{m}}\right] + \frac{1}{M}\left[f_{\mathrm{min}} + (f_{\mathrm{max}} - f_{\mathrm{min}})\eta\right] \\ a_{\mathrm{bz}} = -\frac{Qs_{\mathrm{ref}}}{M}\left[c_{\mathrm{z0}} + c_{\mathrm{za}}\alpha + c_{\mathrm{z}\delta_{\mathrm{m}}}\delta_{\mathrm{m}}\right] \end{cases}$$

- Model reformulation:

$$\begin{bmatrix} \widetilde{f}_1 \\ \widetilde{f}_2 \end{bmatrix} = \begin{bmatrix} \widetilde{g}_{11} & \widetilde{g}_{21} \\ \widetilde{g}_{12} & 0 \end{bmatrix} \cdot \begin{bmatrix} \delta_{\mathrm{m}} \\ \eta \end{bmatrix} \text{ where } \begin{cases} \widetilde{f}_1 = a_{bx} + \frac{Qs_{\mathrm{ref}}}{M}\left[c_{x0} + c_{xa}\alpha\right] - \frac{f_{\mathrm{min}}}{M} \\ \widetilde{f}_2 = a_{bz} + \frac{Qs_{\mathrm{ref}}}{M}\left[c_{z0} + c_{za}\alpha\right] \\ \widetilde{g}_{11} = -\frac{Qs_{\mathrm{ref}}}{M} c_{x\delta_{\mathrm{m}}} \\ \widetilde{g}_{12} = \frac{f_{\mathrm{max}} - f_{\mathrm{min}}}{M} \\ \widetilde{g}_{21} = -\frac{Qs_{\mathrm{ref}}}{M} c_{z\delta_{\mathrm{m}}} \end{cases}$$

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Direct Residual Generation

- Substitute inputs by their computed values δ_{mc} *and* η_{c}

$$\begin{cases} r_{11} = \dfrac{-\widetilde{f}_1+\widetilde{g}_{11}\delta_{\mathrm{mc}}+\widetilde{g}_{21}\eta_{\mathrm{c}}}{\widetilde{g}_{11}} \\ r_{21} = \dfrac{-\widetilde{f}_1+\widetilde{g}_{11}\delta_{\mathrm{mc}}+\widetilde{g}_{21}\eta_{\mathrm{c}}}{\widetilde{g}_{21}} \\ r_{12} = \dfrac{-\widetilde{f}_2+\widetilde{g}_{12}\delta_{\mathrm{mc}}}{\widetilde{g}_{12}} \end{cases}$$

If denominator too close to zero $\rightarrow$ residual r_{ij} not taken into account

- Sensitivity to faults - example

Inject expression of $\widetilde{f}_2$ in residual

$$r_{12} = \frac{-\widetilde{g}_{12}\delta_{\mathrm{m}} + \widetilde{g}_{12}\delta_{\mathrm{mc}}}{\widetilde{g}_{12}} = \delta_{\mathrm{mc}} - \delta_{\mathrm{m}}$$

$\rightarrow$ *identification of the rudder fault*

Additional Residual Generation

- Further combinations between equations
  Here, $\widehat{\delta}_{\mathrm{m}} = \widetilde{f}_2/\widetilde{g}_{12}$ is used in r_{11} and r_{21} to get

$$\begin{cases} \widetilde{r}_{11}^{1} = \dfrac{-\widetilde{f}_1 + \widetilde{g}_{11}\left(\widetilde{f}_2/\widetilde{g}_{12}\right) + \widetilde{g}_{21}\eta_{\mathrm{c}}}{\widetilde{g}_{11}} \\ \widetilde{r}_{21}^{1} = \dfrac{-\widetilde{f}_1 + \widetilde{g}_{11}\left(\widetilde{f}_2/\widetilde{g}_{12}\right) + \widetilde{g}_{21}\eta_{\mathrm{c}}}{\widetilde{g}_{21}} \end{cases}$$

- Sensitivity to faults

Inject expression of $\widetilde{f}_1$ and $\widetilde{f}_2$ in residual

$$\widetilde{r}_{21}^{1} = \frac{-\left(\widetilde{g}_{11}\delta_{\mathrm{m}} + \widetilde{g}_{21}\eta\right) + \widetilde{g}_{11}\left(\widetilde{g}_{12}\delta_{\mathrm{m}}/\widetilde{g}_{12}\right) + \widetilde{g}_{21}\eta_{\mathrm{c}}}{\widetilde{g}_{21}} = \eta_{\mathrm{c}} - \eta$$

$\rightarrow$ *identification of the propulsion fault*

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Simulation set-up

Multiple actuator faults considered

- Propulsion loss: $\forall t > 10\mathrm{s},\ \eta(t) = 0.5\eta_{\mathrm{calc}}(t)$
- Rudder locking-in-place: $\forall t > 15\mathrm{s},\ \delta_{\mathrm{m}}(t) = \delta_{\mathrm{m}}(15\mathrm{s})$

IMU uncertainty

Measurement of q is $\tilde{q} = k_q q + b_q + w_q$

k_q : scale factor, b_q : bias, w_q : Gaussian white noise

Extreme values considered for each measurement

Delay of 2 time steps

Multiplicative model uncertainty

Randomly, each aerodynamic coefficient value is

$c_{\mathrm{sim}} = 0.95c_{\mathrm{model}}$ or $c_{\mathrm{sim}} = 1.05c_{\mathrm{model}}$

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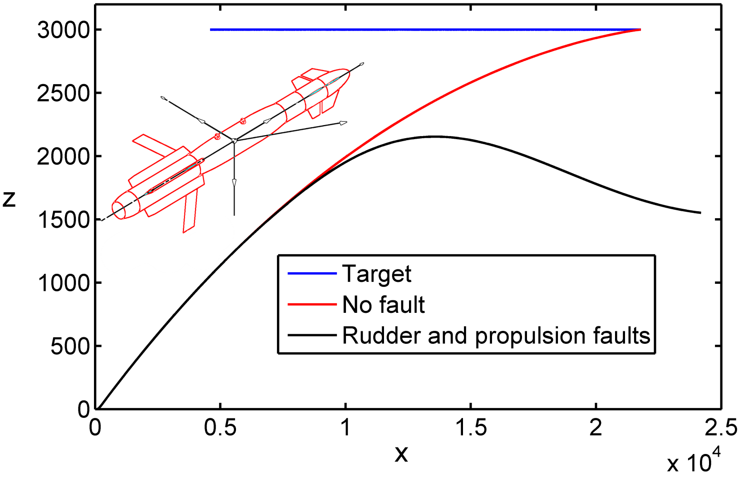
Trajectories



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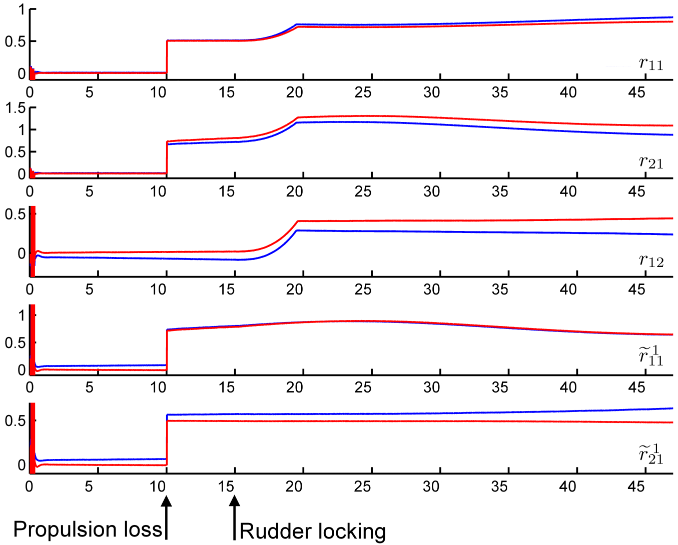
Residuals

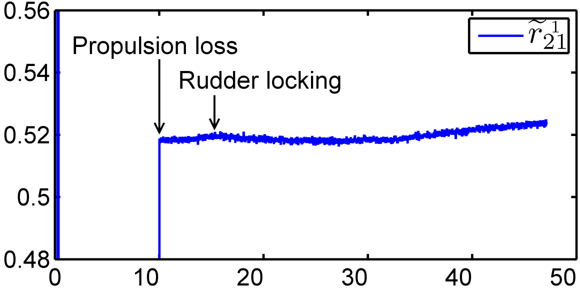


r_{11}
r_{21}
r_{12}
$\tilde{r}_{11}^{1}$
$\tilde{r}_{21}^{1}$
0 5 10 15 20 25 30 35 40 45
Propulsion loss
Rudder locking

Robustness of the residuals

Model error $\widetilde{g}_{11-\mathrm{sim}} = \widetilde{g}_{11-\mathrm{model}} + \varepsilon$, ε small and bounded

$$\widetilde{r}_{21}^{1} = \frac{1}{\widetilde{g}_{21}} \left[\widetilde{g}_{11} \left(-\delta_{\mathrm{m}} + \delta_{\mathrm{m}} - \varepsilon\right) + \widetilde{g}_{21} \left(-\eta + \eta_{\mathrm{c}}\right)\right] = -\frac{\widetilde{g}_{11}}{\widetilde{g}_{21}}\varepsilon\delta_{\mathrm{m}} + \eta_{\mathrm{c}} - \eta$$



Zoom on residual $\widetilde{r}_{21}^{1}$

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Summary and future work

Summary

- Nonlinear FDI scheme applied to a realistic aeronautical model
- Multiple faults detectable, isolable and identifiable
- Static residuals : hard-coding possible, no tuning required
- Acceptable robustness to model and measurement uncertainty
- Formal description of the procedure in the paper
- MAPLE implementation

Future work

- Extend to the 3D case (to be presented at *IEEE SYSTOL 2010*)
- Enhance residual analysis (statistical tests)
- Compare systematically with other FDI approaches

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